CALCULUS TIPS
COMMON MISTAKES!

1. Forgetting to multiply by the chain when taking a derivative.

The chain rule for differentiation says when you take the derivative of an expression which has an inner function, treat the inner function like a simple derivative and multiply by the derivative of the inside. But it is notoriously common to forget to multiply by this derivative!

For example: use the chain rule to differentiate $\ln((3x + 4)^2)$ with respect to $x$. Highlight where you multiply by the chain during the chain rule.

To take this derivative, let $u$ be the inner expression: $u = (3x + 4)^2$.

$$\frac{d}{dx} \ln(u) = \frac{1}{u} \times u' = \frac{1}{(3x+4)^2} \times ((3x + 4)^2)'$$

To find $((3x + 4)^2)'$, we do another chain rule substitution since we have a composition of functions. Let $w$ be the inside expression: $w = 3x + 4$.

$$\frac{d}{dx} (w^2) = 2w \times w' = 2(3x + 4) \times 3$$

So, our final answer is $\frac{1}{(3x+4)^2} \times ((3x + 4)^2)' = \frac{1}{(3x+4)^2} \times 2(3x + 4) \times 3 = \frac{6}{3x+4}$

2. Forgetting to add the constant of integration when taking an indefinite integral.

Remember when you take an indefinite integral, you are finding a family of functions whose derivative is that expression. So if you forget to add a constant $c$, you are restricting yourself to one function of many that could work as your answer! This is particularly important when solving integrals with initial conditions.

Here is a concrete problem showing the importance of the constant:

Evaluate $\int \sin(x)\cos(x) \, dx$ by $u$-substitution in two different ways.

1. $u = \sin(x)$, so $du = \cos(x) \, dx$
   
   So, $\int \sin(x)\cos(x) \, dx = \int u \, du = \frac{u^2}{2} + c$

   $$= \frac{(\sin(x))^2}{2} + c$$

2. $u = \cos(x)$, so $du = -\sin(x) \, dx$

   So, $\int \sin(x)\cos(x) \, dx = \int -u \, du = -\frac{u^2}{2} + c$

   $$= -\frac{(\cos(x))^2}{2} + k$$

Without constants of integration, the answers to the same integral would be unequal. But, considering the constant $k$ can be rewritten as another constant $+ \frac{1}{2}$, we get:

$$-\frac{(\cos(x))^2}{2} + k = \frac{1}{2} - \frac{(\cos(x))^2}{2} + c = \frac{1}{2} - \frac{(\cos(x))^2}{2} + c = \frac{(\sin(x))^2}{2} + c$$, using trigonometry identities. So the answers are the same, and we needed the constant of integration to show this.